VALUING BONDS

A typical corporate bond has:
- a face value of $1,000, which is paid to holder of bond at maturity
- a stated rate of interest (often called the coupon rate), which is a percent of the face value and is paid in semi-annual payments
- a maturity of 10 to 30 or more years - number of years interest is paid

For example, a 20-year, 10% coupon bond that matures in 20 years would:
- Pay 10% of $1,000, or $100, per year in interest, in two installments of $50 every 6 months
- Pay the face value of $1,000 at maturity in 20 years

Thus, the owner of this bond would receive 40 semi-annual interest payments of $50 plus $1,000 at the end of those 40 semi-annual periods (20 years)

The value of the bond, like any asset, is the present value of the future stream of cash benefits to be received. That is, 

\[ VB = \frac{1 - (1+i)^{-n}}{i} \times C + (1+i)^{-n} \times F \]

where, \( C \) = semi-annual coupon payment in $
and \( F \) = Face value of bond ($1,000)
For our 20-year bond, 10% coupon, matures in 20 years:

If Market Demands to Earn 10%, the present value of the future benefits is

\[
VB = \frac{1-(1+.05)^{-40}}{.05} \times 50 + (1+.05)^{-40} \times 1000
\]

\[
VB = 17.159 \times 50 + .14245 \times 1000
\]

\[
VB = 857.95 + 142.05 = $1,000.00
\]

If Market Demands to Earn 12%

\[
VB = \frac{1-(1+.06)^{-40}}{.06} \times 50 + (1+.06)^{-40} \times 1000
\]

\[
VB = 15.046 \times 50 + .09722 \times 1000
\]

\[
VB = 752.23 + 97.22 = $849.53
\]

If Market Demands to Earn 8%

\[
VB = \frac{1-(1+.04)^{-40}}{.04} \times 50 + (1+.04)^{-40} \times 1000
\]

\[
VB = 19.972 \times 50 + .20828 \times 1000
\]

\[
VB = 989.64 + 208.29 = $1,197.93
\]

Bonds that sell below Face sell at a discount
Bond that sell above Face sell at a premium

INTEREST RATE RISK

As Interest Rates Rise - Bond Prices Fall
As Interest Rates Fall - Bond Prices Rise
Long-Term Bonds Prices Fall More Than Short-Term Bond Prices
Bonds have three (3) rates associated with them:

(1) Coupon Rate = percent of face value in interest to be paid each year
(2) Current Yield = (Annual Interest)/Price
    Lets you know the return you receive each year
(3) Yield to Maturity = return earned on bond held until it matures

The return on a bond can come in two ways, either along the way or at maturity. So, the total reward is

\[
YTM = \text{Current Yield(CY)} + \text{Capital Gains/Loss Yield(CGY)}
\]

For a bond selling at a discount: our 10% bond selling to yield 12%

\[
YTM = \frac{100}{849.53} + \text{CGY}
12\% = 11.77 + .23
\]

Thus, the 12% yield to maturity is composed of 11.77% received along the way plus .23% percent in capital gains yield (you get back at maturity $150.47 more than you paid for the bond)

For a bond selling at a premium: our 10% bond selling to yield 8%

\[
YTM = \frac{100}{1197.93} + \text{CGY}
8\% = 8.35 - .35
\]

Thus, the 8% yield to maturity is composed of 8.35% received along the way minus .35% in capital loss yield (you get back at maturity $197.93 less than you paid for the bond)

For a bond selling at face value: our 10% bond selling to yield 10%

\[
YTM = \frac{100}{1000} + \text{CGY}
10\% = 10\% + 0
\]

Thus, the 10% yield to maturity is composed of 10% received along the way plus no capital gains yield (you get back at maturity the same amount that you paid for the bond)
BOND YIELD TO MATURITY

Annual Rate of Return Earned on a Bond if Held Until Maturity

For the 20-year, 10% coupon Bond Selling for $920, Calculate the YTM

\[ 920 = \frac{1-(1+i)^{-40}}{i} \times 50 + (1+i)^{-40} \times 1000 \]

Need to Solve for i, must choose a value for i, plug it into right-hand side and determine if it gives a present value of 920. If not, must raise, or lower, the rate until an i is found that gives the 920 present value.
BOND PRICING THEOREMS

1) Bond prices and bond yields move in opposite directions. As a bond’s yield increases, its price decreases. Conversely, as a bond’s yield decreases, its price increases.

2) For a given change in a bond’s YTM, the longer the term to maturity of the bond, the greater will be the magnitude of the change in the bond’s price.

3) For a given change in a bond’s YTM, the size of the change in the bond’s price increases at a diminishing rate as the bond’s term to maturity lengthens.

4) For a given change in a bond’s YTM, the absolute magnitude of the resulting change in the bond’s price is inversely related to the bond’s coupon rate.

5) For a given absolute change in a bond’s YTM, the magnitude of the price increase caused by a decrease in yield is greater than the price decrease caused by an increase in yield.
BOND PRICE BEHAVIOR

DURATION

A measure of a bond’s sensitivity to changes in bond yields. The original measure is called Macaulay duration.

### Calculating Bond Duration

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<th>Years</th>
<th>Cash Flow</th>
<th>Discount Factor</th>
<th>Present Value</th>
<th>Years × Present Value</th>
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### Modified duration

\[
\text{Modified duration} = \frac{\text{Macaulay duration}}{\left(1 + \text{YTM} / 2\right)}
\]

\[
\% \Delta \text{ in bond price} \approx -\text{Modified duration} \times \Delta \text{ in YTM}
\]