

Chapter 4: Time Value of Money

BASIC KEYS USED IN FINANCE PROBLEMS

The following two key sequences should be done before starting any "new" problem:

~ is used to separate key strokes (3~N: enter 3 then N key)

2nd~Quit: puts calculator in standard mode

2nd~CLR TVM: time value of money worksheet memories

You can also use:

2nd~Reset~Enter (sets all values to default)

Always make sure calculator is set to display two decimals:

2nd~Format~2~Enter: sets calculator to display two decimal places, though calculations in memory will be to thirteen places

Keys for doing time value of money and mortgage problems:

STO~number: there are 10 memories (keys 1 through 9)

RCL~number: recalls amount in memory

+/-: key changes sign of value on screen (positive to negative)

N: number of cash flows or payments

I/Y: annual interest rate

PV: starting amount or loan amount

PMT: period cash flow or loan payment

FV: ending amount

CPT: compute answer (solve for missing variable)

2nd~P/Y: number of payments per year
2nd~C/Y: number of compounding periods per year
2nd~BGN: makes all cash flows occur at end of time period
2nd~AMORT: shows amortization worksheet for TVM or mortgage problems and includes P1 (starting payment), P2 (ending payment),
Down arrow: for moving through amortization menu
P1: Payment 1
P2: Payment 2
BAL: remaining balance on loan after P2
PRN: principle amount paid during period P1 to P2
INT: interest paid during period P1 to P2
CF: used for entering period cash flows
CE/E: clear entry on screen

FUTURE VALUE (FV) OF SINGLE AMOUNT - What an amount will grow to over time

Deposit \$1,000 into bank that pays 10% interest

$$FV(1) = 1,000 + 1,000(.10) = 1,000(1.10) = 1,100$$

$$FV(2) = 1,000(1.10) + 1,000(1.10)(.10) \\ = 1,000(1.10)(1.10) = 1,000(1.10)^2 = 1,210$$

$$FV(3) = 1,000(1.10)^2 + 1,000(1.10)^2(.10) \\ = 1,000(1.10)^2(1.10) = 1,000(1.10)^3 = 1,331$$

$$FV(n) = PV(1+i)^n$$

After Year	Accumulation	
1	1,100	100
2	1,210	100 + 10
3	1,331	100 + 10 + 10 + 1

\$5,000 deposited in bank paying 6% will earn after 8 years:

$$FV = 5,000(1+.06)^8 = 5,000(1.59384807) = 7,969.24$$

CALCULATOR: Future value of \$1,000 after three years earning 10% per year

Key Strokes:

2nd~Quit

2nd~CLR TVM

2nd~P/Y~1~Enter~CE/C

3~N

10~I/Y

1000~PV

CPT~FV [Answer: 1,331.00]

PRESENT VALUE (PV) OF A SINGLE AMOUNT -What you must start with to reach a future amount

$$FV(n) = PV(1+i)^n$$

$$PV = FV/(1+i)^n$$

$$PV = FV(1+i)^{-n}$$

How much do you need to deposit today so you will have \$10,000 accumulated after 7 years if you can earn 5% per year?

$$PVS = 10,000(1+.05)^{-7} = 10,000(.710681233) = 7,106.81$$

CALCULATOR: Present value, or amount you need to deposit today, so you will have accumulated \$7,106.81 three years from now earning 5% per year

Key Strokes:

2nd~Quit

2nd~CLR TVM

2nd~P/Y~1~Enter+CE/C

10~N

5~I/Y

7106.81~FV

CPT~PV [Answer: 10,000.00]

MORE FREQUENT COMPOUNDING OF A SINGLE AMOUNT

The more often interest is added to your savings, the sooner you can start earning interest on your interest.

Assume \$1,000 invested at 10% per year:

$$FV(1) = 1,000(1.10)^1 = 1,100 \quad [\text{one year is one time period}]$$

$$FV(2) = 1,000(1+.10/2)^2 = 1,102.50$$

[one year is 2 six-month time periods: semi-annual]

$$FV(4) = 1,000(1+.10/4)^4 = 1,103.81$$

[one year is 4 three-month time periods: quarterly compounding]

$$FV(12) = 1,000(1+.10/12)^{12} = 1,104.71$$

[one year is 12 one-month time periods: monthly compounding]

$$FV(365) = 1,000(1+.10/365)^{365} = 1,105.55$$

[one year is 365 daily time periods: daily compounding]

Effective Annual Rate

$$EAR = (1 + i/m)^m - 1$$

The accumulation for a single initial investment with frequent compounding is

$$FVS = PV \times (1+i/m)^{mn}$$

n = total number of years

CALCULATOR: Bank pays interest of 4%, compounded quarterly. If you deposit \$2,000 and leave it in the bank for four years, how much will you have accumulated?

Key Strokes:

2nd~Quit

2nd~CLR TVM

2nd~P/Y~1~Enter

Down Arrow~4~Enter+CE/C

4~N

4~I/Y

2000~PV

CPT~FV [Answer: 2,345.16]

ANNUITY - a series of equal periodic cash flows

FUTURE VALUE OF AN ANNUITY - what a series of equal periodic cash flows will grow to

0	1	2	3	
	1,000	1,000	1,000	
				$1,000(1.10)^0 = 1,000(1.00) = 1,000$
				$1,000(1.10)^1 = 1,000(1.10) = 1,100$
				$1,000(1.10)^2 = 1,000(1.21) = \underline{1,210}$
				Total Accumulation = 3,310

$$FVA = A \times \frac{(1+i)^n - 1}{i} = 1,000 \times \frac{(1.10)^3 - 1}{.10}$$
$$= 1,000 \times 3.31 = 3,310$$

You deposit \$2,000 each year for 10 years into a bank that pays 6% per year. How much will you have accumulated at the end of 10 years?

Key Strokes:

2nd~Quit

2nd~CLR TVM

2nd~P/Y~1~Enter (C/Y is changed to P/Y entry automatically)

10~N

6~I/Y

2000~PMT

CPT~FV [Answer: 26,361.59]

PRESENT VALUE OF AN ANNUITY - the starting amount that will create a series of equal periodic cash flows.

0		1	2	3
		1,000	1,000	1,000
909.09 =	1,000(.90909) =	1,000(1.10) ⁻¹	_____	
826.44 =	1,000(.82644) =	1,000(1.10) ⁻²	_____	
<u>751.31</u> =	<u>1,000(.75131) =</u>	<u>1,000(1.10)⁻³</u>	_____	
2,486.84 =	Starting amount needed			

$$PVA = A \times \frac{1 - (1+i)^{-n}}{i} = 1,000 \times \frac{1 - (1.10)^{-3}}{.10}$$

$$= 1,000 \times 2.48684 = 2,486.84$$

CALCULATOR: What amount do you need to deposit today so you can withdraw \$1,000 per year for three years while earning 6%.

Key Strokes:

2nd~Quit

2nd~CLR TVM

2nd~P/Y~1~Enter (C/Y is changed to P/Y entry automatically)

10~N

6~I/Y

1000~PMT

CPT~PV [Answer: 2,486.85]

MORE FREQUENT COMPOUNDING - ANNUITY

To calculate the future or present value of an annuity with more frequent compounding:

First compute the effect annual rate (EAR)

$$\text{EAR} = (1+i/m)^m - 1$$

Then use the EAR in the place of (i) in the FVA and PVA formulas

Problem: Find the future value of \$1,000 per year for 3 years if interest is 6%, compounded monthly.

$$\text{EAR} = (1+.06/12)^{12} - 1 = 1.06167781 - 1 = .06167781$$

$$\text{FVA} = \frac{(1+.06167781)^3 - 1}{.06167781} \times 1000 = 3,188.83$$

CALCULATOR:

Key Strokes:

2nd~Quit

2nd~CLR TVM

2nd~P/Y~1~Enter

Down Arrow~12~Enter~C/CE

3~N

6~I/Y

1000~PMT

CPT~FV [Answer: 3,188.84]

MORTGAGES - mortgages are simply present value of annuity problems

$$\text{LOAN} = \text{PMT} \times \frac{1 - (1+i/12)^{-n}}{i/12}$$

LOAN = amount borrowed

PMT = monthly payment

n = number of monthly payments to repay loan

i = annual state rate of interest

Consider a mortgage of \$100,000 to be repaid monthly over 30 years at an interest rate of 7%

To calculate the monthly payment on a loan or mortgage:

$$\text{LOAN} = \text{PMT} \times \frac{1 - (1+i/12)^{-n}}{i/12}$$

$$\text{PMT} = \frac{\text{LOAN}}{\frac{1 - (1+i/12)^{-n}}{i/12}}$$

$$\begin{aligned} \text{PMT} &= \frac{\text{LOAN}}{\frac{1 - (1+i/12)^{-n}}{i/12}} = \frac{100,000}{\frac{1 - (1+.07/12)^{-360}}{.07/12}} \\ &= \frac{100,000}{150.3075679} = 665.31 \end{aligned}$$

CALCULATOR: Mortgage of \$100,000 at 7% for 30 years, compute the monthly payment.

2nd~Reset~Enter~CE/C

360~N

7~I/Y

100,000~+/-~PV

CPT~PMT [Ans: \$665.30]

PAYOFF OR REMAINING BALANCE AT ANY POINT IN TIME

$$RB_n = PMT \times \frac{1 - (1+i/12)^{-n}}{i/12}$$

How much will you owe after paying on loan for 5 years? (that means 25 years left)

$$RB_{300} = 665.31 \times \frac{1 - (1+.07/12)^{-300}}{.07/12}$$

$$= 665.31 \times 141.4869034 = 94,132.66$$

CALCULATOR: Amount owed after 5 years on 30 year mortgage of \$100,000 at 7%.

2nd~Amort

60~Enter

Down Arrow~60~Enter

Down Arrow [Answer: 94,131.76]

INTEREST PAID DURING ANY TIME INTERVAL

CALCULATOR: How much interest will be paid on the loan during the fifth year? Calculate the remaining balance at the start of the fifth year and at the end of the fifth year:

$$RBBeg = PO_{312} = 665.31 \times \frac{1 - (1 + .07/12)^{-312}}{.07/12} = 95,474.55$$

$$RBEnd = PO_{300} = 665.31 \times \frac{1 - (1 + .07/12)^{-300}}{.07/12} = 94,131.59$$

Principal paid during the fifth year =

$$95,474.55 - 94,131.59 = 1,342.96$$

Total amount paid during the fifth year

$$665.31 \times 12 = 7,983.72$$

Since the 7,983.72 must go toward paying interest and paying off the loan (principal)

Total payments - amount toward principal = interest paid

$$7,983.72 - 1,342.96 = 6,640.76 \text{ interest paid during year 5}$$

Total interest paid over life of loan =

Total payments - amount borrowed

$$665.31 \times 360 - 100,000 = 239,511.60 - 100,000 = 139,511.60$$

CALCULATOR: Interest paid during the 5th year on a 30 year mortgage of \$100,000 at 7%.

2nd~Amort

49~Enter

60~Enter

Down Arrow [Balance end of 5th year: 94,131.76]

Down Arrow [Principal during 5th year: 1,342.91]

Down Arrow [Interest during 5th year: 6,640.69]