Chapter 23

INTEREST-RATE FUTURES CONTRACTS

FUTURES CONTRACT - contract to sell (deliver) or buy (take delivery of) a standardized quantity (or dollar amount) of an asset on a set date (settlement date)

COMMODITY FUTURES

Agriculture
Metals
Energy

FINANCIAL FUTURES

Foreign Currencies
Debt Instruments
Stock Indexes

FUTURES EXCHANGES

FUTURES CLEARINGHOUSE

TRADING

Short Position
Long Position
Margin
Mark to Market
Margin Maintenance
Minimum Price Change
Daily Maximum Price Change
Open Interest
Settlement Price
BASIS

Basis = Cash Price - Futures Price

HEDGING

Short Hedge
Long Hedge

SPECULATING

Leverage

HEDGING EXAMPLE:

Corn: 5,000 bushel contract

Cash (Spot) Price = 2.01/bu
Dec Futures Price = 2.31/bu

Farmer will have 5,000 bushels to harvest and sell in December
Kellogg will need to buy 5,000 bushels for manufacturing in December

Farmer has corn (long position) and needs to sell (short position) it in December

    Farmer is long corn and needs to go short in the Futures Contract

    Farmer sells (goes short) one December contract

Kellogg needs corn (short position) and needs to buy (long position) in December

    Kellogg is short corn and needs to go long in the Futures contract

    Kellogg buys (goes long) one December contract
At expiration, Futures price must equal Spot price

Assume corn is selling at end of December at:

1) 2.31/bu (December spot price)

Farmer buys one contract to close position

Sold @ 2.31 and bought @ 2.31  Profit = .00
Final price = 2.31 + .00 = 2.31/bu

Kellogg sells one contract to close position

Bought @ 2.31 and sold @ 2.31  Profit = .00
Final price = 2.31 + .00 = 2.31/bu

2) 2.81/bu (December spot price)

Farmer buys one contract to close position

Sold @ 2.31 and bought @ 2.81  Loss = -.50
Final price = 2.81 - .50 = 2.31/bu

Kellogg sells one contract to close position

Bought @ 2.31 and sold @ 2.81  Profit = .50
Final price = 2.81 - .50 = 2.31/bu

3) 1.81/bu (December spot price)

Farmer buys one contract to close position

Sold @ 2.31 and bought @ 1.81  Profit = .50
Final price = 1.81 + .50 = 2.31/bu

Kellogg sells one contract to close position

Bought @ 2.31 and sold @ 1.81  Loss = .50
Final price = 1.81 + .50 = 2.31/bu
Treasury Bill Futures

Traded on International Money Market (IMM)

13-week T-bills
$1 million face value
Deliver can be in newly issued or seasoned

Quoted in bank discount

\[ Y_d = \frac{100 - \text{index price}}{100} = \frac{100 - 92.52}{1000} = 7.48\% \]

Discount  =  \( Y_d \times 1,000,000 \times t/360 \)

\[ = 0.0748 \times 1,000,000 \times 91/360 = 18,907.78 \]

Invoice Price = \$1,000,000 - 18,908.78 = \$981,092.22

Minimum tick is .01 = \$25.28

Treasury Bond Futures

CBOT

$100,000 face value
20-year, 8% coupon
Prices quoted in percent of face in 32nds
Minimum tick is 1/32 or \$31.25

Deliverables must have at least 15 year to maturity
Conversion factors for approved deliverables
Cheapest-to-deliver Issue (implied repo rate)
Timing option - seller has choice of time during deliver month when to deliver
Seller can also choose to deliver after close (8:00 pm).
Buyer is never sure of which Treasury will be delivered or when
Pricing and Arbitrage in Futures Markets

Law of one price - financial asset must have same price regardless of the means by which it is created. Assets cannot sell for different prices in different markets

Consider 20-year, 12% bond selling at par
Consider futures contract selling at 107 and settles in three months
Consider money can be borrowed at 8%

Sell contract at 107
Buy bond at 100
Borrow 100 for three months at 8%

At settlement:
Sell bond for 107
Accrued interest is 3
Total proceeds 110

Repay loan 100
Interest 2
Total outlay 102

Profit 8

This strategy is called a cash-and-carry trade
Consider 20-year, 12% bond selling at par
Consider futures contract selling at 92 and settles in three months
Consider money can be lent at 8%

Buy contract at 92
Sell bond short for 100
Invest proceeds at 8%

At settlement:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy bond for</td>
<td>92</td>
</tr>
<tr>
<td>Accrued interest</td>
<td>3</td>
</tr>
<tr>
<td>Total outlay</td>
<td>95</td>
</tr>
<tr>
<td>Principal from loan</td>
<td>100</td>
</tr>
<tr>
<td>Interest from loan</td>
<td>2</td>
</tr>
<tr>
<td>Total proceeds</td>
<td>102</td>
</tr>
<tr>
<td>Profit</td>
<td>7</td>
</tr>
</tbody>
</table>

This strategy is called a reverse cash-and-carry trade

What is arbitrage-free contract price? 99
Theoretical Futures Price

Determined by:
  Price of bond in cash market
  Coupon rate on bond
  Interest rate for borrowing and lending until settlement data

Let,  \( r = \) lending/borrowing rate
  \( c = \) current yield on bond
  \( P = \) cash price of bond
  \( F = \) futures price
  \( t = \) time to settlement

Cash and carry:

<table>
<thead>
<tr>
<th>Price of bond</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accrued interest</td>
<td>ctP</td>
</tr>
<tr>
<td>Total proceeds</td>
<td>( F + ctP )</td>
</tr>
<tr>
<td>Repayment of loan</td>
<td>P</td>
</tr>
<tr>
<td>Interest on loan</td>
<td>rtP</td>
</tr>
<tr>
<td>Total outlay</td>
<td>( P + rtP )</td>
</tr>
</tbody>
</table>

Profit = \( F + ctP \) - (\( P + rtP \))

In equilibrium, the profit should be zero, therefore:

\[
0 = F + ctP - (P + rtP) \\
F = P[1 + t(r - c)]
\]

For the 12% deliverable bond, with financing rate at 8%, and \( t = 0.25 \)

\[
F = 100[1 + 0.25(0.08 - 0.12)] = 99
\]
r - c is the cost of carry

if \( r > c \) is positive means current yield greater than financing cost
Futures price will sell at a discount to cash price
if \( r < c \) is negative means cost of financing is greater than current yield
Futures price will sell at a premium to cash price

Arbitrage model assumptions not considered:
- Interim cash flows (bond coupon payment)
- Variation margin
- Lending versus Borrowing rate differences
- Deliverable Bond is not known
- Delivery date not known

Applications to Bond Portfolio Management

Speculation
- Expect rising rates: short futures
- Expect falling rates: long futures

Controlling Interest Rate Risk

Approximate number of contracts to portfolio duration

\[
\text{# contracts} = \frac{(D_T - D_I)P_I}{D_F P_F}
\]

\( D_T \) = target effective duration
\( D_I \) = initial effective duration
\( P_I \) = initial market value of portfolio
\( D_F \) = effective duration for futures contract
\( P_F \) = market value of futures contract

If \( D_T > D_I \) than buy futures contracts
If \( D_T < D_I \) than sell futures contracts
Synthetic Securities

Own a 20-year Treasury bond
Sell a Treasury futures contract that settles in three months
Have created a risk-free 3-month Treasury

This synthetic return should equal the yield on 3-month Treasuries

RSP = riskless short term position
CBP = cash bond position
FBP = bond futures position

\[
\text{RSP} = \text{CBP} - \text{FBP} \quad \text{(cash bond and short futures give short term yield)}
\]

\[
\text{CBP} = \text{RSP} + \text{FBP} \quad \text{(cash bond is created by buying futures and T-bill)}
\]

\[
\text{FBP} = \text{CBP} - \text{RSP} \quad \text{(futures created cash bond and shorting T-bill)}
\]

Hedging

Perfect hedge
Basis risk
Cross hedge

Short hedge used to protect against price decline
Long hedge used to protect against price increase

Hedge Ratio - engaging in cross hedge

\[
\text{Hedge Ratio} = \frac{\text{volatility of bond to be hedged}}{\text{volatility of hedging instrument}}
\]