Yield Curve

Spot Rate Curve - yield on Treasury zeros at each maturity

Because there are no Treasury zeros beyond one year, must derive a spot rate curve

Resulting curve is a theoretical spot rate curve

Use on-the-run Treasuries
Use one-the-run Treasuries and off-the-run
Use all Treasuries
Use Treasury coupon strips

When using on-the-run Treasuries, there are only a few available, so must interpolate linearly

Instead of interpolating, use bootstrapping

YTM for 6-month and 1-year Treasury is also their spot rates

Valuing Bond

Need spot rates for each future semi-annual time period

Interest rate models are generally arbitrage-free models that are based on how short-term rates can evolve over time

Interest Rate Lattice - binomial tree

Future rates can be one of two possible rates, with equal likelihood
Rates evolve over time randomly – lognormal random walk
Each node is one time period - one year

\[ \sigma = \text{volatility of one-year forward rate} \]

\[ r_{1,H} = r_{1,L} (e^{2\sigma}) \]

\[ r_{2,HH} = r_{2,LL} (e^{4\sigma}) \]
\[ r_{2,HL} = r_{2,LL} (e^{2\sigma}) \]

Value at first node is the average of the present values of the cash flows at the next set of nodes

To find forward rates must use an iterative process. Assume two-year, 4% coupon bond:

1) select value for \( r_{1L} \) (assume 4.5%)
2) find value for \( r_{1H} \) based on \( r_{1L} \) and \( \sigma \)
3) compute value of bond one year from now
   a) first compute value two year from now. for two year bond that is \$1040
   b) calculate present value of \$1040 using higher rate = \( VH_1 \)
   c) calculate present value of \$1040 using lower rate = \( VL_1 \)
   d) add coupon for period one to values of \( VH_1 \) and \( VL_1 \) to get period one cash flows
   e) calculate present value of two cash flow from period one using the one-year forward rate, which is known
4) take average of two present values
5) compare average in step (4) to actual price of bond
6) if computed value (average) equals price, trial assumed rate is correct forward rate. If not, must try different rates until one is found that makes computed value equal to actual value

Can now extrapolate out to three year bond to find the second forward rate

For callable bond, when the issues is exercisable, the bond value at a node must be change to reflect the lesser of its value if not called and the call price