A typical corporate bond has:

- a face value of $1,000, which is paid to the holder of the bond at maturity
- a stated rate of interest (often called the coupon rate), which is a percent of the face value and is paid in semi-annual payments
- a maturity of 10 to 30 or more years - number of years interest is paid

For example, a 20-year, 10% coupon bond would:
- Pay 10% of $1,000, or $100, per year in interest, in two installments of $50 every 6 months
- Pay the face value of $1,000 at maturity in 20 years

Thus, the owner of this bond would receive 40 semi-annual interest payments of $50 plus $1,000 at the end of those 40 semi-annual periods (20 years)

The value of the bond, like any asset, is the present value of the future stream of cash benefits to be received. That is,

\[
VB = \frac{1-(1+i)^n}{i} \times C + \frac{(1+i)^n}{i} \times F
\]

where, C = semi-annual coupon payment in $
and F = Face value of bond ($1,000)
For our 20-year bond, 10% coupon:

If Market Demands to Earn 10%, the present value of the future benefits is

\[ VB = \frac{1-(1+.05)^{-40}}{.05} \times 50 + (1+.05)^{-40} \times 1000 \]
\[ VB = 17.159 \times 50 + .14245 \times 1000 \]
\[ VB = 857.95 + 142.05 = $1,000.00 \]

If Market Demands to Earn 12%

\[ VB = \frac{1-(1+.06)^{-40}}{.06} \times 50 + (1+.06)^{-40} \times 1000 \]
\[ VB = 15.046 \times 50 + .09722 \times 1000 \]
\[ VB = 752.23 + 97.22 = $849.53 \]

If Market Demands to Earn 8%

\[ VB = \frac{1-(1+.04)^{-40}}{.04} \times 50 + (1+.04)^{-40} \times 1000 \]
\[ VB = 19.972 \times 50 + .20828 \times 1000 \]
\[ VB = 989.64 + 208.29 = $1,197.93 \]

Bonds that sell below Face sell at a discount
Bond that sell above Face sell at a premium
BOND YIELD COMPOSITION

Bonds have three (3) rates associated with them:

(1) Coupon Rate = percent of face value in interest to be paid each year
(2) Current Yield = (Annual Interest)/Price
    Lets you know the return you receive each year
(3) Yield to Maturity = return earned on bond held until it matures

The return on a bond can come in two ways, either along the way or at maturity. So, the total reward is

\[ \text{YTM} = \text{Current Yield(CY)} + \text{Capital Gains/Loss Yield(CGY)} \]

For a bond selling at a discount: our 10% bond selling to yield 12%

\[ \frac{100}{849.53} + \text{CGY} = 12\% \]
\[ 11.77 + .23 \]

Thus, the 12% yield to maturity is composed of 11.77% received along the way plus .23% percent in capital gains yield (you get back at maturity $150.47 more than you paid for the bond)

For a bond selling at a premium: our 10% bond selling to yield 8%

\[ \frac{100}{1197.93} + \text{CGY} = 8\% \]
\[ 8.35 - .35 \]

Thus, the 8% yield to maturity is composed of 8.35% received along the way minus .35% in capital loss yield (you get back at maturity $197.93 less than you paid for the bond)

For a bond selling at face value: our 10% bond selling to yield 10%

\[ \frac{100}{1000} + \text{CGY} = 10\% \]
\[ 10\% + 0 \]
Thus, the 10% yield to maturity is composed of 10% received along the way plus no capital gains yield (you get back at maturity the same amount that you paid for the bond)

For premium bonds:

\[ \text{coupon rate} > \text{current yield} > \text{YTM} \]

For discount bonds:

\[ \text{coupon rate} < \text{current yield} < \text{YTM} \]

For par value bonds:

\[ \text{coupon rate} = \text{current yield} = \text{YTM} \]
BOND YIELD TO MATURITY

Annual Rate of Return Earned on a Bond if Held Until Maturity

For the 20-year, 10% coupon Bond Selling for $920, Calculate the YTM

\[ 920 = \frac{1-(1+i)^{-40}}{i} \times 50 + (1+i)^{-40} \times 1000 \]

Need to Solve for \( i \), must choose a value for \( i \), plug it into right-hand side and determine if it gives a present value of 920. If not, must raise, or lower, the rate until an \( i \) is found that gives the 920 present value.
BOND PRICING THEOREMS

1) Bond prices and bond yields move in opposite directions. As a bond’s yield increases, its price decreases. Conversely, as a bond’s yield decreases, its price increases.

2) For a given change in a bond’s YTM, the longer the term to maturity of the bond, the greater will be the magnitude of the change in the bond’s price.

3) For a given change in a bond’s YTM, the size of the change in the bond’s price increases at a diminishing rate as the bond’s term to maturity lengthens.

4) For a given change in a bond’s YTM, the absolute magnitude of the resulting change in the bond’s price is inversely related to the bond’s coupon rate.

5) For a given absolute change in a bond’s YTM, the magnitude of the price increase caused by a decrease in yield is greater than the price decrease caused by an increase in yield.
BOND PRICE BEHAVIOR

DURATION

\[
\text{Modified duration} = \frac{\text{Macaulay duration}}{\left(1 + \frac{\text{YTM}}{2}\right)}
\]

A measure of a bond’s sensitivity to changes in bond yields. The original measure is called Macaulay duration.

\%Δ in bond price ≈ −Modified duration×Δ in YTM
BOND PRICE BEHAVIOR

DURATION

A measure of a bond’s sensitivity to changes in bond yields. The original measure is called Macaulay duration.

<table>
<thead>
<tr>
<th>Years</th>
<th>Cash Flow</th>
<th>Discount Factor</th>
<th>Present Value</th>
<th>Years × Present Value + Bond Price</th>
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$1,000.00 Bond Price $2,7259 years Bond Duration

\[
\text{Modified duration} = \frac{\text{Macaulay duration}}{\left(1 + \frac{\text{YTM}}{2}\right)}
\]

\[
\% \Delta \text{ in bond price} \approx -\text{Modified duration} \times \Delta \text{ in YTM}
\]
Immunization

• Used to protect a bond portfolio against interest rate risk
  – Price risk and reinvestment risk cancel
• Price risk results from relationship between bond prices and rates
• Reinvestment risk results from uncertainty about the reinvestment rate for future coupon income
Immunization

• Risk components move in opposite directions
  – Favorable results on one side can be used to offset unfavorable results on the other

• Portfolio immunized if the duration of the portfolio is equal to investment horizon
  – Like owning zero-coupon bond
DEMONSTRATION OF HOW DURATION AND INTEREST RATE RISK ARE RELATED

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<tr>
<th>Invest</th>
<th>FV factor at 10% for</th>
<th>FV</th>
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<td>2.7355 yr</td>
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<td>1000 1.29787 1297.87</td>
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10% 3 years

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<th>Cashflow</th>
<th>PV factor</th>
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<th>PV x t</th>
<th>PV/(PVxt)</th>
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</table>

Immediately after purchase rate increases to 12%

| 100 | 1.217362 121.7362 |
| 100 | 1.08693   108.693  |
| 1100| 0.970473 1067.521 |

1297.95