Our goal in this chapter is to understand the relationship between bond prices and yields, and to examine some of the fundamental tools of bond risk analysis used by fixed-income portfolio managers.
Bond Basics

**Straight bond**

An IOU that obligates the issuer to pay to the bondholder a fixed sum of money (called the principal, par value, or face value) at the bond’s maturity, along with constant, periodic interest payments (called coupons) during the life of the bond.

- U.S. Treasury bonds are straight bonds.
- Special features may be attached, creating convertible bonds, “putable” bonds, etc.
Bond Basics

• Two basic yield measures for a bond are its *coupon rate* and *current yield*.

Coupon rate = \( \frac{\text{Annual coupon}}{\text{Par value}} \)

Current yield = \( \frac{\text{Annual coupon}}{\text{Bond price}} \)
Straight Bond Prices and Yield to Maturity

Yield to maturity (YTM)
The discount rate that equates a bond’s price with the present value of its future cash flows.
Straight Bond Prices and Yield to Maturity

Bond price = present value of all the coupon payments + present value of the principal

Bond price = \[ \frac{C}{YTM} \left[ 1 - \frac{1}{\left(1 + \frac{YTM}{2}\right)^{2M}} \right] + \frac{FV}{\left(1 + \frac{YTM}{2}\right)^{2M}} \]

where \( C \) = annual coupon, the sum of 2 semiannual coupons
\( FV \) = face value
\( M \) = maturity in years
Premium and Discount Bonds

• Bonds are commonly distinguished according to the relative relationship between their selling price and their par value.

• *Premium bonds*: price > par value
  YTM < coupon rate

• *Discount bonds*: price < par value
  YTM > coupon rate

• *Par bonds*: price = par value
  YTM = coupon rate
Premium and Discount Bonds

The chart illustrates the relationship between bond prices and time to maturity. Bond prices are shown in percentage of par value, with 'Premium' indicating prices above par, 'Par' at par value, and 'Discount' indicating prices below par. The graph shows how bond prices change over time as they approach maturity.
Premium and Discount Bonds

• In general, when the coupon rate and YTM are held constant …

for discount bonds: the longer the term to maturity, the greater the discount from par value, and

for premium bonds: the longer the term to maturity, the greater the premium over par value.
Relationships among Yield Measures

• Since the current yield is always between the coupon rate and the yield to maturity (unless the bond is selling at par) …

  for premium bonds:
  coupon rate > current yield > YTM

  for discount bonds:
  coupon rate < current yield < YTM

  for par value bonds:
  coupon rate = current yield = YTM
To obtain current information on Treasury bond prices and yields, try the search tool at:

- [http://www.bondsonline.com](http://www.bondsonline.com)
Calculating Yields

• To calculate a bond’s yield given its price, we use the straight bond formula and then try different yields until we come across the one that produces the given price.

\[
\text{Bond price} = \frac{C}{\text{YTM}} \left[ 1 - \frac{1}{\left(1 + \frac{\text{YTM}}{2}\right)^{2M}} \right] + \frac{\text{FV}}{\left(1 + \frac{\text{YTM}}{2}\right)^{2M}}
\]

• To speed up the calculation, financial calculators and spreadsheets may be used.
Yield to Call

• A *callable bond* allows the issuer to buy back the bond at a specified *call price* anytime after an initial *call protection period*, until the bond matures.
Yield to Call

• *Yield to call (YTC)* is a yield measure that assumes a bond issue will be called at its earliest possible call date.

\[
\text{Callable bond price} = \frac{C}{YTC} \left[ 1 - \frac{1}{(1 + \frac{YTC}{2})^{2T}} \right] + \frac{CP}{(1 + \frac{YTC}{2})^{2T}}
\]

where
- \( C = \) constant annual coupon
- \( CP = \) call price of the bond
- \( T = \) time in years to earliest possible call date
- \( YTC = \) yield to call assuming semiannual coupons
Interest Rate Risk

**Interest rate risk**

The possibility that changes in interest rates will result in losses in a bond’s value.

• The yield actually earned or “realized” on a bond is called the *realized yield*, and this is almost never exactly equal to the *yield to maturity*, or *promised yield*.
Interest Rate Risk and Maturity
Malkiel’s Theorems

① Bond prices and bond yields move in opposite directions. As a bond’s yield increases, its price decreases. Conversely, as a bond’s yield decreases, its price increases.

② For a given change in a bond’s YTM, the longer the term to maturity of the bond, the greater will be the magnitude of the change in the bond’s price.
Malkiel’s Theorems

③ For a given change in a bond’s YTM, the size of the change in the bond’s price increases at a diminishing rate as the bond’s term to maturity lengthens.

④ For a given change in a bond’s YTM, the absolute magnitude of the resulting change in the bond’s price is inversely related to the bond’s coupon rate.
Malkiel’s Theorems

⑤ For a given absolute change in a bond’s YTM, the magnitude of the price increase caused by a decrease in yield is greater than the price decrease caused by an increase in yield.
Malkiel’s Theorems

### Bond Prices and Yields

<table>
<thead>
<tr>
<th>Yields</th>
<th>Time to Maturity</th>
<th>5 Years</th>
<th>10 Years</th>
<th>20 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>7%</td>
<td></td>
<td>$1,041.58</td>
<td>$1,071.06</td>
<td>$1,106.78</td>
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<tr>
<td>9%</td>
<td></td>
<td>960.44</td>
<td>934.96</td>
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<tr>
<td>Price difference</td>
<td></td>
<td>$ 81.14</td>
<td>$ 136.10</td>
<td>$ 198.79</td>
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</tbody>
</table>

### Twenty-Year Bond Prices and Yields

<table>
<thead>
<tr>
<th>Yields</th>
<th>6 Percent</th>
<th>8 Percent</th>
<th>10 Percent</th>
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<tr>
<td>6%</td>
<td>$1,000.00</td>
<td>$1,231.15</td>
<td>$1,462.30</td>
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<tr>
<td>8%</td>
<td>802.07</td>
<td>1,000.00</td>
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<tr>
<td>10%</td>
<td>656.82</td>
<td>828.41</td>
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</table>
Duration

A measure of a bond’s sensitivity to changes in bond yields. The original measure is called *Macaulay duration*.

\[
\% \Delta \text{ in bond price} \approx -\text{Duration} \times \left( \frac{\Delta \text{ in YTM}}{1 + \text{YTM}/2} \right)
\]

→ Two bonds with the same duration, but not necessarily the same maturity, will have approximately the same price sensitivity to a (small) change in bond yields.
Duration

Modified duration = \frac{\text{Macaulay duration}}{\left(1 + \frac{\text{YTM}}{2}\right)}

So,

\%\Delta \text{ in bond price} \approx -\text{Modified duration} \times \Delta \text{ in YTM}
Calculating Macaulay’s Duration

• Macaulay’s duration values are stated in years, and are often described as a bond’s effective maturity.

• *For a zero coupon bond*, duration = maturity.

• *For a coupon bond*, duration = a weighted average of individual maturities of all the bond’s separate cash flows, where the weights are proportionate to the present values of each cash flow.
Calculating Macaulay’s Duration

<table>
<thead>
<tr>
<th>Years</th>
<th>Cash Flow</th>
<th>Discount Factor</th>
<th>Present Value</th>
<th>Years × Present Value ÷ Bond Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$ 40</td>
<td>.96154</td>
<td>$ 38.4615</td>
<td>.0192 years</td>
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<td>1</td>
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<td>36.9822</td>
<td>.0370</td>
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<td>35.5599</td>
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<td></td>
<td>$1,000.00</td>
<td>2.7259 years</td>
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<tr>
<td></td>
<td></td>
<td>Bond Price</td>
<td>Bond Duration</td>
<td></td>
</tr>
</tbody>
</table>
Calculating Macaulay’s Duration

• In general, for a bond paying constant semiannual coupons,

\[
\text{Duration} = \frac{1 + \frac{\text{YTM}}{2}}{\text{YTM}} - \frac{1 + \frac{\text{YTM}}{2} + M(C - \text{YTM})}{\text{YTM} + C \left[ \left( 1 + \frac{\text{YTM}}{2} \right)^{2M} - 1 \right]}
\]

where $C$ = constant annual coupon rate
$M$ = bond maturity in years
YTM = yield to maturity assuming semiannual coupons
Calculating Macaulay’s Duration

• If a bond is selling for par value, the duration formula can be simplified:

\[
\text{Par value bond duration} = \frac{1 + YTM}{2} \left[ 1 - \frac{1}{\left(1 + \frac{YTM}{2}\right)^{2M}} \right]
\]
Properties of Duration

① All else the same, the longer a bond’s maturity, the longer is its duration.

② All else the same, a bond’s duration increases at a decreasing rate as maturity lengthens.

③ All else the same, the higher a bond’s coupon, the shorter is its duration.

④ All else the same, a higher yield to maturity implies a shorter duration, and a lower yield to maturity implies a longer
Properties of Duration
Dedicated Portfolios

Dedicated portfolio
A bond portfolio created to prepare for a future cash outlay, e.g. pension funds. The date the payment is due is commonly called the portfolio’s target date.
Work the Web

For a practical view of bond portfolio management, visit:
  ▶ http://www.jamesbaker.com
Reinvestment Risk

Reinvestment rate risk

The uncertainty about future or target date portfolio value that results from the need to reinvest bond coupons at yields not known in advance.

- A simple solution is to purchase zero coupon bonds. In practice however, U.S. Treasury STRIPS are the only zero coupon bonds issued in sufficiently large quantities, and they have lower yields than even the highest quality corporate bonds.
Price Risk versus Reinvestment Rate Risk

**Price risk**
The risk that bond prices will decrease. Arises in dedicated portfolios when the target date value of a bond or bond portfolio is not known with certainty.

- Interest rate increases act to decrease bond prices (price risk) but increase the future value of reinvested coupons (reinvestment rate risk), and vice versa.
Immunization

- It is possible to engineer a portfolio such that price risk and reinvestment rate risk offset each other more or less precisely.

- Constructing a portfolio to minimize the uncertainty surrounding its target date value.
Immunization by Duration Matching

- A dedicated portfolio can be immunized by *duration matching* - matching the duration of the portfolio to its target date.
- Then the impacts of price and reinvestment rate risk will almost exactly offset, and interest rate changes will have a minimal impact on the target date value of the portfolio.
Immunization by Duration Matching
Dynamic Immunization

**Dynamic immunization**

Periodic rebalancing of a dedicated bond portfolio to maintain a duration that matches the target maturity date.

- The advantage is that the reinvestment risk caused by continually changing bond yields is greatly reduced.
- The drawback is that each rebalancing incurs management and transaction costs.
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